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A simple model for optical capture of atoms in strong magnetic quadrupole fields

D. Haubrich, A. Höpe and D. Meschede

Institut für Quantenoptik, Universität Hannover, Welfengarten 1, 30167 Hannover, Germany

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The radiative capture of cesium atoms from the gas phase in a magnetooptic trap with strong magnetic field gradients is studied. A simplified analytic model is used to derive an upper limit for capture velocities. The resulting scaling law agrees well with the observed number of atoms and with the density as a function of field gradient.

1. Introduction

The capture of slow neutral atoms by radiation pressure forces directly from the gas phase in a magnetic quadrupole field has become an important method for the generation and application of gaseous samples of very cold atoms. In the standard configuration of such a magnetooptic trap (MOT) [1–3] three pairs of counterpropagating laser beams with $\sigma^+\sigma^-$ circular polarization intersect at the zeropoint of a magnetic quadrupole field. The spectral atomic resonance properties are spatially modulated by the magnetic field. The corresponding variation of the radiation pressure forces provides confinement through a net binding force towards the center of the quadrupole for sufficiently slow atoms and furthermore influences the rate at which atoms with low velocities are slowed and caught in the trap.

For many applications of trapped samples of cold atoms, for instance the investigation of very high densities when at low energy the de Broglie wavelength of atoms becomes larger than their separation [4] or the generation of localized atoms in strong magnetic traps [5], an efficient loading of the trap and hence a thorough understanding of the capture process is desirable. At low magnetic field gradients of order 10 G/cm detailed experimental investigations have been reported [6,7]. While up to now analytic estimates generally neglect the role of the magnetic field, many aspects of the observed properties of the MOT can be satisfactorily explained from numerical simulations. A full theoretical understanding taking into account the three-dimensional nature of the system remains to be given. Even in one dimension the lorentzian dependence of the radiation pressure force on atomic velocity and position through magnetic detuning results in a complicated nonlinear equation of atomic motion for the capture process in a trap. Analytic estimates [7,8] have consequently been successful when this dependence was negligible. This is no longer the case in a strong gradient environment since the forces vary rapidly even for a slowly moving atom.

To overcome this difficulty we have theoretically studied in a one-dimensional model the scattering of atoms with initial velocity v_0 into outbound states with velocity $v_0 - \delta v$, which is analytically accessible through a perturbative treatment. A capture velocity can be defined from the breakdown of the perturbation approach and a theoretical scaling law for the dependence on magnetic field gradient is inferred.

We have also measured the atomic number and density in a strong gradient MOT and compared theory and experimental results. The experimental setup is outlined in sec. 3.

2. An upper limit for capture velocities

The equilibrium number N_S of atoms in a vapor pressure magnetooptic trap is determined by a balance of loading slow atoms from the background gas

Table 1
Magnetooptic parameters of alkali atoms.

	Li	Na	Rb	Cs
$\gamma/2\pi$ (MHz)	5.85	9.89	5.89	5.30
$a_0 = \hbar k\gamma/2m$ (m/s^2)	1.58×10^6	9.15×10^5	1.11×10^5	5.87×10^4
$\kappa = 2\pi/\lambda\gamma$ (m/s)$^{-1}$	0.25	0.17	0.22	0.22
$\beta/b = \mu_B/\hbar\gamma$ (cm^{-1})	0.24	0.14	0.24	0.26
I_0 (mW/cm^2)	2.54	6.33	1.62	1.10

against losses mostly due to collisions with "hot" thermal atoms [2]. This can be described by a simple differential equation

$$\mathrm{d}N/\mathrm{d}t = R_L - N/\tau, \tag{1}$$

where R_L is the loading rate of the trap and $1/\tau = n\sigma\bar{v}$ is the loss rate of atoms in the trap by collisions with background atoms (n: density of atoms, σ: collision cross section). We have neglected higher order terms in N since we will be concerned with relatively low densities of atoms [9]. The steady state number N_S of atoms in the trap is then given by $N_S = R_L\tau$. It is straightforward to determine τ from the loading process with $N(t) = N_S[1 - \exp(-t/\tau)]$.

The loading rate R_L is given by the number of atoms entering the absorption volume enclosed by a surface S and with velocities $v \leqslant v_{cap}$ which is generally much smaller than $\bar{v} = (2k_BT/m)^{1/2}$, the most probable velocity. From the maxwellian distribution of velocities one finds [2]:

$$R_L \approx \tfrac{1}{2}nS\bar{v}\,(v_{cap}/\bar{v})^4\,. \tag{2}$$

Both the capture velocity v_{cap} and the effective surface S of the "trap absorption" volume are functions of the quadrupole field. The loss rate $1/\tau$ on the other hand should be relatively insensitive to an increasing gradient $b = \mathrm{d}B/\mathrm{d}z$ since magnetic energies are generally small in the trap. However, a weak variation of the collision cross section σ may be indicated by the slow increase of the time constant τ from 0.17 s at 63 G/cm to 0.22 s at 140 G/cm which we observe in trap loading at constant background pressure $p = 5\times10^{-8}$ mbar. A similar behaviour was also reported by Gibble et al. [7]. We will neglect this slow dependence in the following.

The radiation pressure force has a lorentzian frequency dependence with a saturated linewidth $\Delta\omega_{fwhm} = \gamma\sqrt{1 + s_0}$ ($s_0 = I/I_0$, I_0: saturation intensity). Due to the Doppler effect the center of the lorentzian depends on the velocity v and due to Zeeman tuning on the position z in the quadrupole field. The corresponding classical acceleration a_+ (a_-) describes the action of a σ^+ (σ^-) polarized wave travelling in the positive (negative) z-direction:

$$a_\pm(v,z) = \pm\frac{a_0 s_0}{1+s_0}\left(1 + \frac{4[\Delta \mp (\kappa v + \beta z)]^2}{1+s_0}\right)^{-1}. \tag{3}$$

The maximum acceleration is $a_0 = \hbar k\gamma/2m$ and the laser detuning $\Delta = (\omega - \omega_0)/\gamma$ is measured in units of the linewidth γ. The Doppler shift parameter is $\kappa = 2\pi/\lambda\gamma$, and $\beta = (m_{F_2}g_{F_2} - m_{F_1}g_{F_1})\mu_B b/\hbar\gamma$ is the Zeeman shift parameter (g_F: Landé g-factor) which reduces to $\beta = \mu_B b/\hbar\gamma$ for transitions between extreme components with $m_F = F$. A list of these parameters for several alkali atoms is given in table 1.

This model is valid only for a $J = 0\rightarrow1$ transition while most atoms of interest have a rich magnetic fine and hyperfine structure. Slow atoms, however, will quickly be oriented by optical pumping to a maximum m_F state along the local magnetic field axis and hence very much resemble the $J = 0\rightarrow1$ transition. This assumption is not valid for confined atoms interacting with several trapping beams as is well known from the observation of polarization gradient cooling [10,11]. Consequently, our model is applicable to the capture process only.

While the atom is transiting the center of the quadrupole the velocity change in a time interval $\mathrm{d}t$ is

$$\mathrm{d}v = [a_+(v,z) + a_-(v,z)]\,\mathrm{d}t, \tag{4}$$

where we can replace $\mathrm{d}t$ by $\mathrm{d}z/v$. The nonlinear differential equation (4) cannot be solved analytically

and hence it is not possible to directly derive a maximum initial velocity for which an atom would come to rest in a certain trap volume, an approach that was possible for the low gradient estimates of refs. [7,8].

Fast atoms with $v \gg v_{cap}$ will experience a small velocity change δv only. This small variation, $v(t \to \infty) = v_0 - \delta v$ can be approximately calculated through a perturbative treatment of eq. (4). Once the solution of this approximation is known we define a capture velocity from the breakdown of the perturbative approach, i.e. when $\delta v/v_0$ is no longer small. Consequently our model places an upper limit on capture velocities which nevertheless reflects the dependence on experimental parameters such as the magnetic field gradient, hence yielding suitable scaling laws.

The resulting differential equation to first order in δv is

$$\frac{\mathrm{d}}{\mathrm{d}z}\delta v = \frac{\alpha}{v_0}\left[(a_+ + a_-)\left(1 - \frac{\delta v}{v_0}\right) + \frac{8\kappa\delta v}{1+s_0}\left[a_+^2(\Delta - \kappa v_0 - \beta z) + a_-^2(\Delta + \kappa v_0 + \beta z)\right]\right]. \tag{5}$$

This expression can be integrated by standard techniques with the introduction of an integrating factor.

Before proceeding to the full formal solution let us consider a further simplified model to gain some physical understanding: for the case of large detuning ($\Delta \gg 1$) we can replace the non-overlapping lorentzian acceleration profiles centered at $z_\pm = -\beta^{-1}(\kappa v \mp \Delta)$ by rectangular acceleration profiles of height $\alpha = a_0 s_0/(1+s_0)$ in the range $(z - z_\pm)^2 \leqslant (1+s_0)/4\beta^2 =: (\Delta z_{1/2}/2)^2$. For small velocity changes the atom then passes the resonance halfwidth $\Delta z_{1/2} = \sqrt{1+s_0}/\beta$ in a time interval $\Delta t_0 = \Delta z_{1/2}/v_0$ and the velocity variation is $\Delta v \approx \alpha \Delta t_0 = \alpha\sqrt{1+s_0}/\beta v_0$.

The slowing process in the quadrupole $\sigma^+\sigma^-$-configuration is a consequence of an acceleration (a_+) and a subsequent deceleration (a_-) (fig. 1). A net slowing force is obtained, since during acceleration the effective time interval is shortened to

$$\Delta t_1 = \Delta z_1/v_0 \approx \Delta t_0\left(1 - \frac{\kappa\Delta v}{2\sqrt{1+s_0}}\right),$$

while it is prolonged during deceleration to

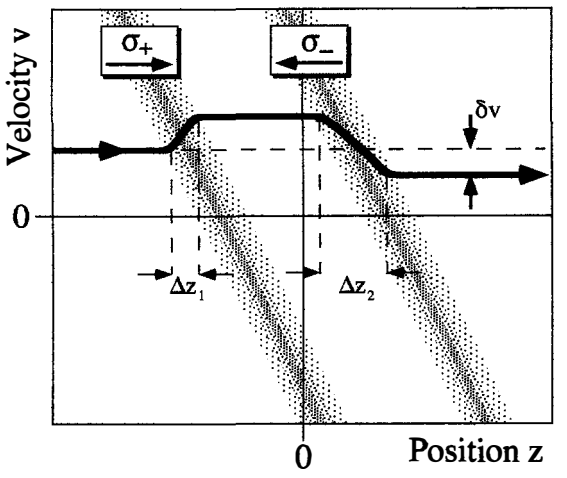

Fig. 1. Atomic trajectory in phase space for a one-dimensional magnetooptic trap. The intensity of radiative forces is indicated by the shaded areas.

$$\Delta t_2 = \Delta z_2/v_0 \approx \Delta t_0\left(1 + \frac{\kappa\Delta v}{2\sqrt{1+s_0}}\right).$$

As a result we find a differential relative velocity change:

$$\frac{\delta v}{v_0} \approx \frac{\alpha}{v_0}(\Delta t_1 - \Delta t_2) = -\frac{a_0^2 s_0^2}{(1+s_0)^{3/2}}\frac{\kappa}{\beta^2}\frac{1}{v_0^3}. \tag{6}$$

From eq. (6) and ref. [2] it is clear that only particles with low initial velocity will loose enough kinetic energy to be caught in the trap. We therefore define an upper limit for v_{cap} by demanding that for $v_0 \leqslant v_{cap}$ the velocity change should exceed $|\delta v/v_0| \geqslant 1/2$. The numerical value does not play a significant role for the scaling law.

The full formal analysis of eq. (5) shows that eq. (6) has to be multiplied by $4\pi\Delta^2/(1+s_0+4\Delta^2)$ which accounts for a vanishing friction force at zero laser detuning Δ as expected. For v_{cap} one finds to first order in δv:

$$v_{cap} = \left(\frac{a_0^2 s_0^2 \kappa}{(1+s_0)^{3/2}}\right)^{1/3}\left(\frac{8\pi\Delta^2}{1+s_0+4\Delta^2}\right)^{1/3}\beta^{-2/3}. \tag{7}$$

The real situation is three-dimensional, of course, and we can no longer assume that an atom flies through

the center of the trap. Instead, we have to consider the more general case where an atom at $v \leqslant v_{cap}$ is moving in the z-direction with an impact parameter ρ in the xy-plane.

From the work by Lindquist et al. [6] and Steane et al. [12] it seems clear that the slowing process acts mainly in a radial direction and hence the specific local combination of laser polarization, quadrupole field, and atomic orientation does not seem to play a very significant role other than determining overall forces. Therefore in eq. (3) one should replace the magnetic tuning factor βz by $\beta[(\rho/2)^2 + z^2]^{1/2}$. Even for slow atoms with impact parameters $\rho > |\Delta/\beta|$ magnetic detuning at stronger field gradients remains always so large that maximum deceleration is no longer possible. This condition defines an upper limit for a capture radius: $\rho_{cap} = |\Delta/\beta|$ beyond which atoms cannot be caught in the trap. From this estimate we conclude that in our case the resonance volume usually has an effective radius smaller than the trapping beams, for instance $\rho_{cap} \leqslant 0.8$ mm for $\Delta = -2$, $b = 100$ G/cm. Hence in our stiff magnetooptic trap the trapping surface, or perhaps better the geometric trap absorption cross section, varies roughly as $S \leqslant 4\pi\rho_{cap}^2 \sim b^{-2}$ where for the estimate we neglect the difference in the z- and in the ρ-direction.

In a low field MOT, on the other hand, the capture radius becomes larger than typical laboratory laser beams. At such large radii the capture cross section is no longer limited by magnetic detuning but by the radius of the trapping laser beams. This limiting case was studied in detail by Lindquist et al. [6] and Gibble et al. [7] who found the number of trapped atoms to be proportional to the trapping laser beam cross section.

One would expect to increase the trap absorption cross section with larger Δ. However, enhanced Δ also reduces the friction coefficient [13,14], and hence the time it takes an atom to relax from its stopping position to the confinement volume will rapidly increase so that abortion of the capture process becomes more and more likely.

From our experiments we find that the optimum detuning for maximum trap fluorescence is largely insensitive to the magnetic field gradient at saturation parameters s_0 of order 1–10, and furthermore occurs near $\Delta \sim \Omega_R$ (Ω_R: Rabi frequency), i.e. $\Delta \sim \sqrt{s_0}$ which is shown in (fig. 2). Disregarding the slow variation of the collision cross section σ with field gradient b we can theoretically give an upper limit for the number of trapped atoms from $N_S = R_L\tau$:

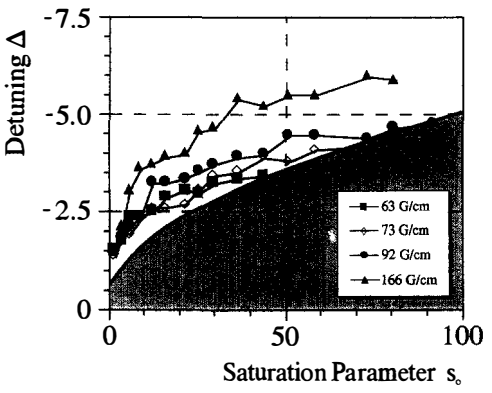

Fig. 2. Detuning for maximum fluorescence intensity of the trap. Solid line represents $\Delta = \sqrt{s_0}$ where the detuning equals the Rabi frequency.

$$N_S \leqslant 2\pi \frac{\rho^2 v_{cap}}{\sigma} \left(\frac{v_{cap}}{\bar{v}}\right)^4 . \tag{8}$$

As pointed out earlier v_{cap} varies as $\beta^{-2/3}$ and for stronger field gradients we have $\rho_{cap} \sim \beta^{-1}$. Insertion into eq. (8) yields a rapid scaling of N_S with field gradient at constant detuning Δ and saturation s_0:

$$N_S \sim \beta^{-14/3} . \tag{9}$$

In fig. 3, a comparison of this dependence with our data is given. In the upper right corner the distribution of the numerical fitted exponents of all our data is shown in a histogram. The comparison with the computed value shows good agreement. Note that the curves link our range of parameters roughly to the values that other authors find in standard MOT situations [3,6].

For the density of trapped atoms we experimentally find a weaker dependence on the field gradient. For constant laser parameters Δ and s_0 we observe a linear dependence of the radius of the trapped cloud of atoms on the inverse magnetic field gradient: $r_{trap} \sim 1/b$. This observation can be interpreted by a temperature of the trapped sample determined by Δ and s_0 only [5]. Consequently the density should scale as $n \sim N_S r_{trap}^{-3}$ or

$$n \sim \beta^{-5/3} . \tag{10}$$

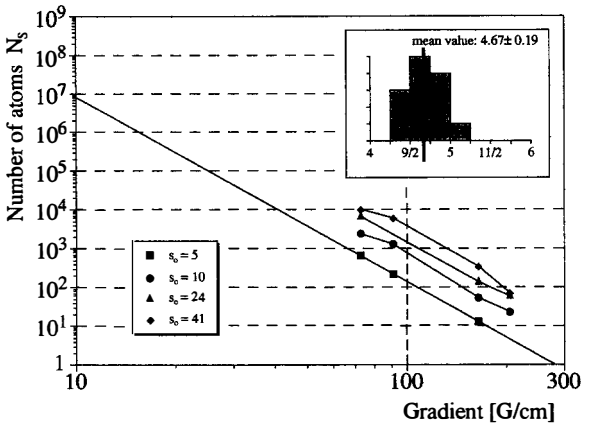

Fig. 3. Steady state number of atoms N_S as a function of magnetic field gradient $b = dB/dz$. Detuning is set for maximum fluorescence. The histogram in the upper right corner shows the distribution and the mean value of the fitted exponents. The solid line extrapolates our data for low saturation intensities to the regime of low b. For such low field MOTs numerous authors have found atom numbers of 10^7 or more.

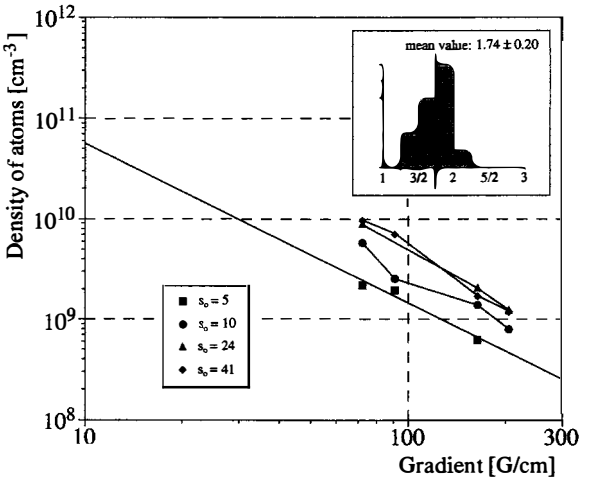

Fig. 4. Density of atoms n as a function of magnetic field gradient $b = dB/dz$. The histogram shows the distribution and the mean value of the fitted exponents. Using the $b^{-5/3}$ scaling law for n the solid line once again extrapolates to values found by other authors.

This expression matches our experimental data quite well (fig. 4). It provides additional evidence for our interpretation since it relies on a second independent measurement, the volume of the trap.

3. Experimental setup

We operate our MOT with trapping laser beam intensities ranging from $I = 2$–$100\ \mathrm{mW/cm^2}$ and a $1/e^2$ beam diameter of 7 mm in each arm. The light is provided by a Ti:sapphire laser and guided to the experiment via an optical fiber. Optical quality cell windows were AR-coated to facilitate equilibration of the laser beam intensities. Low magnetic field gradients up to $b = 25$ G/cm are obtained from two coils in "anti-Helmholtz" configuration. By replacing the coils with permanently magnetized and oppositely oriented discs we reach gradients from $b = 60$ G/cm to more than 300 G/cm depending on the distance and on the number of discs on each side. To measure the detuning from resonance we observe the heterodyne beat note between the Ti:sapphire laser and a diode laser stabilized to a saturated absorption line of the cesium $6s_{1/2}$, $F = 4 \rightarrow 6p_{3/2}$, $F' = 5$ transition.

The number of atoms is determined from the fluorescence intensity imaged onto a Peltier cooled CCD camera. At low atom numbers and hence low fluorescence levels a detection and measurement of the cloud of atoms was only possible by means of the CCD camera due to spatial resolution.

Since we cannot switch off the permanent magnetic field it is necessary to estimate the influence of the magnetic field on this measurement. In a properly aligned trap, the atoms are collected in the vicinity of the zero field center of the magnetic quadrupole. This condition was tested by switching off the trapping light beams which leaves the atoms in a purely magnetic trap, because gravitational forces are overcome for field gradients $b > 25$ G/cm. After 50 ms the spatial distribution of the atoms was probed by strobing the trapped cloud with a 5 ms laser pulse. It was always found to be centered at the same location as the MOT, apart from a small sagging which we attribute to gravity. For "properly aligned clouds", the center of the fluorescent ball not only coincides with the center of the magnetically trapped sample, its position furthermore varies less than 30 μm with laser intensity up to the highest light levels ($100\ \mathrm{mW/cm^2}$). The only observation here is a blow-up of the radius of the cloud with light intensity. Also the observation of sub-Doppler temperatures [5] for the trapped atoms shows that the trap is indeed operated at the magnetic zero point. To be specific, at the fringes of

the trapped samples the magnetic field strength does not exceed 0.7 G at 90 μm trap radius and 73 G/cm field gradient, and 0.4 G at 20 μm radius and 207 G/cm gradient, respectively. The magnetic shift does hence not exceed 1 MHz which is less than 1/5 of the natural line width.

We have measured atom numbers ranging from about 10^4 at 73 G/cm down to as few as 10 atoms at high gradients whereas the density of the trapped atoms varies between 9×10^9 cm^{-3} down to about 5×10^8 cm^{-3} only. The smallest trap radius we observed is $r \approx 20$ μm which is limited by our imaging ratio and the pixel size of the camera.

Comparable measurements at smaller gradients would have required the increase of laser beam diameters beyond our viewports to satisfy the condition that the capture radius is smaller than the radius of the beams.

4. Conclusion

Maximum capture velocities for standard MOTs under consideration of the magnetic field have usually been determined from numerical simulations [6]. We have instead analytically studied a simplified one-dimensional model yielding an upper limit with relatively weak dependence $v_{cap} \sim b^{-2/3}$ of the capture velocity on magnetic field gradient.

The moderate reduction of capture velocity and geometric capture cross section translates into a much stronger reduction of the loading rate, $R_L \sim S v_{cap}^4 \sim b^{-14/3}$. The observed numbers of atoms N_S follow this scaling law surprisingly closely and furthermore link our values to those obtained with standard low field MOTs. The strong reduction in N_S is accompanied by a moderate decrease in the density $n = N_S/V_{trap} \sim b^{-5/3}$.

For higher field gradients we calculate an effective capture radius ρ_{cap} much smaller than the radius of the trapping beams. This means that in stiffer traps a magnified beam diameter does not lead to an increase in the number of trapped atoms, because ρ_{cap} is an upper limit for slowing atoms in the trap. For moderate saturation parameters s_0 we observe maximum trap fluorescence always near detunings $\Delta \sim \Omega_R \sim \sqrt{s_0}$.

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